

# Cooperation in prebiotic evolution: The effect of small populations of molecules

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**Abstract.** In the prebiotic context, before the emergence of the first forms of life, the introduction of cooperation in molecular evolution allowed replicators, molecules capable of self-replication, to form increasingly complex systems, culminating in the emergence of the first cells. However, it is not known how cooperation was introduced in this context. To understand this process, the competition between two types of replicators was analyzed: one that is selfish and another that is cooperative, that is, that catalyzes the replication of other replicators. Therefore, it was studied whether a small population of cooperative replicators can survive if introduced into a population dominated by selfish replicators, in order to analyze the effect of cooperation on competition between pre-biotic replicators. This competition was described by the replicator equation model. Two methodological approaches were used in this study: one analytical and the other computational. In the analytical approach, the equilibrium points of the system of differential equations of the studied model and the conditions for the local stability of these points were found. In the computational approach, two types of simulations were made: one deterministic and the other stochastic. The deterministic simulation was carried out based on traditional methods of numerical solution of differential equations and is more representative for large populations. The stochastic simulation was carried out using the Gillespie algorithm and is more representative for small populations. It is concluded that, in small total populations, susceptibility to random variations is more pronounced, reducing the chance of survival of cooperative replicators in most conditions. Conversely, even for small total populations, cooperative replicators can avoid extinction under conditions that would normally lead to extinction if the total population were larger. Furthermore, it was observed that the catalysis precision of cooperative replicators is a decisive factor: the greater the precision, the smaller the initial population of cooperative replicators needed so that the probability of extinction is less than 50%.

**Keywords.** Prebiotic Evolution, Molecular Evolution, Evolutionary Dynamics, Evolution of Cooperation, Replicator Equation, Stochastic simulation, Gillespie algorithm.

## 1. Introduction

Life is a phenomenon that has fascinated many generations of thinkers since the emergence of philosophy and science [1]. Despite this enormous interest that persists throughout the history of human thought, it is still not known how life originated. However, there are some clues about the best ways to understand the origin of life [2]. One of these paths consists of the self-organization of matter [3]. In this context, one scenario to study the emergence of life is the prebiotic evolution.

As the name suggests, “pre” refers to the word “before” and “biotic” refers to the word “life”, that is, “prebiotic evolution” refers to evolution that occurred before the emergence of life. But which entities evolved if there was no life? For an entity to evolve, it needs to be able to reproduce, that is, transmit essential information to the upcoming generations [4]. In the prebiotic context, these evolving entities were self-replicating molecules, which can be called replicators [5].

The interactions between the replicators and the environment determined which replicators would

survive and which would become extinct [5]. But what were the forms of interaction between these prebiotic replicators? The first form is intrinsically related to the process of natural selection: competition [6]. Given that there were finite resources for replicators to replicate in the environment, they competed for these resources and the fittest replicated more, passing on their biological information to the next generations [6]. However, in the prebiotic context, there were no precise replication mechanisms, such that the mutation rate was high enough to prevent the formation of complex molecules, a problem known as the Eigen Paradox [7]. A solution to this difficulty was obtained through the introduction of hypercycles, which are cooperative replicators that catalyze each other's replication [8]. Thus, cooperation is another form of interaction between prebiotic replicators, allowing the formation of increasingly complex systems [9].

However, an intriguing question arises: in the prebiotic context, in which the environment was highly competitive, dominated by non-cooperative replicators, how did cooperation manage to be introduced into molecular evolution? The objective of this work is to study the possible answers to this question through the analysis of the competition between populations of non-cooperative and cooperative replicators.

## 2. Research Methods

First, we studied the replicator equation model for the case of prebiotic evolution, which describes the time evolution of replicator populations through differential equations derived from evolutionary game theory [10].

Through an analytical study of this model, the equilibrium points of the system of differential equations studied and the conditions for the local stability of these points are found. Based on the conditions found, general conclusions are drawn about this model.

Through a computational study of this model, both deterministic and stochastic simulations are carried out, which were made using authorial programs written in the Python language. The deterministic simulation, which represents the results obtained for large populations, is made using traditional methods of numerical solution of differential equations. The stochastic simulation, which represents the results for small populations, is carried out using the Gillespie algorithm [11].

The results of the computer simulations are presented in graphs, more specifically, in heat maps. By analyzing these graphs, the results of these two types of simulations are compared, and their biological meanings are evaluated.

Finally, these results are discussed in relation to the results obtained by other studies with different approaches found in the scientific literature.

## 2.1 System modeling

One way to study how the introduction of cooperation occurred in prebiotic evolution is through the analysis of competition between two types of replicators: one that is selfish and another that is cooperative [10]. A replicator is cooperative if it catalyzes the replication of another replicator, and is selfish if it does not catalyze it.

We assume that population 1 corresponds to the population of selfish replicators and population 2 corresponds to the population of cooperative replicators. The equations that describe the time evolution of these populations of replicators are

$$\dot{X}_1 = X_1(r_1 + B_1X_2 - \psi), \quad (1)$$

$$\dot{X}_2 = X_2((r_2 - C) + B_2X_2 - \psi), \quad (2)$$

where  $X_i$  is the number of replicators in population  $i$ ,  $r_i$  is the growth rate of population  $i$ ,  $C$  is the cost of catalyzing the replication of other replicators, and  $B_i$  is the rate with which the replication of replicator  $i$  is catalyzed [10].

Analyzing (1) and (2), it is observed that the time variation rates  $\dot{X}_i$  of the populations are proportional to the sizes of the populations  $X_i$ , which is due to self-replication. Furthermore, catalysis is modeled by the product of population sizes. It can be seen that (1) has the term  $B_1X_2$  because it is considered that, when cooperative replicators emerged, their catalysis mechanisms were not yet very precise, in such a way that they could catalyze the replication of selfish replicators with which they competed. Furthermore, as population 2, of cooperative replicators, catalyzes the replication of other replicators, this population bears the catalysis cost  $C$ .

The term  $\psi$  introduces competition between the two populations and represents natural selection in this model. It is obtained from the constant population condition  $X_1 + X_2 = X_T$  (differentiating this expression and using (1) and (2)), where  $X_T$  is the total population, which is given by

$$\psi = \frac{r_1X_1 + (r_2 - C)X_2 + B_1X_1X_2 + B_2X_2^2}{X_T}, \quad (3)$$

The assumption of a constant population can be justified by the hypothesis that, in the prebiotic context, replicators emerged in environments with sufficient resources to maintain life, but with limited sizes, which were able to support a fixed number of replicators [12].

## 2.2 Analytical study

Replacing the expression of  $\psi$  given by (3) in the equations (1) and (2), we obtain that

$$\dot{X}_1 = r_1X_1 + B_1X_1X_2 - \frac{r_1X_1^2 + (r_2 - C)X_1X_2 + B_1X_1^2X_2 + B_2X_1X_2^2}{X_T}, \quad (4)$$

$$\dot{X}_2 = (r_2 - C)X_2 + B_2X_2^2 - \frac{r_1X_1X_2 + (r_2 - C)X_2^2 + B_1X_1X_2^2 + B_2X_2^3}{X_T}, \quad (5)$$

To analytically study the time evolution of the populations of replicators described by the differential equations (4) and (5), the local stability of the equilibrium points is analyzed (in which the populations are constant after elapsed a long time) [13], which are found when solving the system of equations  $\dot{X}_1 = 0$  and  $\dot{X}_2 = 0$ .

To analyze the local stability of the equilibrium points of this system of differential equations, the system is linearized around these points, calculating the Jacobian matrix [13], which is given by

$$J = \begin{pmatrix} \frac{\partial X_1}{\partial X_1} & \frac{\partial X_1}{\partial X_2} \\ \frac{\partial X_2}{\partial X_1} & \frac{\partial X_2}{\partial X_2} \end{pmatrix}, \quad (6)$$

Subsequently, the eigenvalues  $\lambda$  of the Jacobian matrix are determined for each equilibrium point  $(X_1^*, X_2^*)$  [13], according to

$$\det(J(X_1^*, X_2^*) - \lambda I) = 0, \quad (7)$$

where  $I$  is the identity matrix.

The local stability of the equilibrium points is analyzed based on the sign of the eigenvalues [13]. If an eigenvalue has a negative real part, then it is stable, that is, the population tends to approach this point in equilibrium. If an eigenvalue has a positive real part, then it is unstable, that is, the population tends to move away from this point in equilibrium. If the eigenvalues have different signs, then this is a saddle point, that is, in certain directions it is an attractor and in others it is a repeller. If an eigenvalue has an imaginary part, the population exhibits oscillatory behavior at equilibrium.

Using the procedures described above, the equilibrium points  $(X_1^*, X_2^*)$  of the system of equations  $\dot{X}_1 = 0$  and  $\dot{X}_2 = 0$  are calculated, which are  $(0, 0)$ ,  $(X_T, 0)$ ,  $(0, X_T)$ ,  $(0, \frac{C-r_2}{B_2})$  e  $(\frac{r_2+B_2X_T-C-r_1-B_1X_T}{B_2-B_1}, \frac{C+r_1-r_2}{B_2-B_1})$ . Analyzing the stability of these points, the following results are obtained:

1. The point  $(0,0)$  is not physical due to the constant population condition  $X_1 + X_2 = X_T$ ;
2. The point  $(X_T, 0)$  is unstable if  $r_2 > C + r_1$ , which means that a small population of cooperative replicators can invade a large population of selfish replicators if the growth rate of the population of cooperative replicators is higher than the growth rate of the population of selfish replicators added to the cost suffered by the population of cooperative replicators for catalyzing the replication of other replicators;
3. The point  $(0, X_T)$  is unstable if  $r_2 + B_2X_T < C + r_1 + B_1X_T$ , which means that a small population of selfish replicators can invade a large population of cooperative replicators if the net growth rate

(population growth rate plus the total beneficial effect of catalysis) of the population of cooperative replicators is lower than the net growth rate of the population of selfish replicators plus the catalysis cost suffered by the population of cooperative replicators;

4. The point  $(0, \frac{C-r_2}{B_2})$  is not physical due to the constant population condition  $X_1 + X_2 = X_T$ ;
5. The point  $(\frac{r_2+B_2X_T-C-r_1-B_1X_T}{B_2-B_1}, \frac{C+r_1-r_2}{B_2-B_1})$  is physical for  $B_1 > B_2$  and  $r_2 > C + r_1$  and  $r_2 + B_2X_T < C + r_1 + B_1X_T$  or for  $B_2 > B_1$  and  $r_2 < C + r_1$  and  $r_2 + B_2X_T > C + r_1 + B_1X_T$ . For the first case, the coexistence point is always stable, such that it corresponds to conditions 2 and 3 above, which means that the populations of selfish and cooperative replicators coexist in equilibrium. For the second case, the coexistence point is always unstable, in such a way as to correspond to the opposite of conditions 2 and 3 above, which means that the populations of selfish and cooperative replicators do not coexist in equilibrium, that is, one of them goes extinct.

### 3. Results and discussions

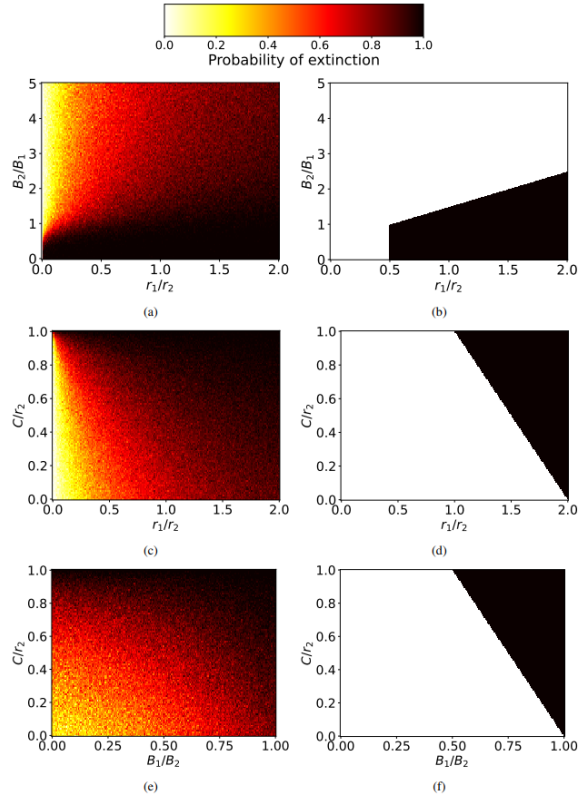
In the analytical study, it was observed that, as condition 3 indicates, for large total populations, it is enough for the catalysis rate of the cooperative replicators  $B_2$  to be a little higher than that of the selfish replicators  $B_1$  (being the case for the prebiotic context) so that the population of cooperative replicators is not invaded by selfish replicators. Thus, the most interesting question that will be analyzed is how cooperative replicators managed to be introduced into a population dominated by selfish replicators. To achieve this, the initial conditions of the computational simulations carried out are always the same (except for those simulations that vary the initial conditions): the initial population of cooperative replicators is small and the initial population of selfish replicators is large.

Therefore, several heat maps were made to analyze the relationship between the probability of extinction of the population of cooperative replicators and the different parameters of the model studied, in such a way that all combinations of parameters were explored in depth. Furthermore, for each combination of parameters, two heat maps were created: one through deterministic simulation (representing the result for a large total population) and another through stochastic simulation (representing the result for a small total population), which was done with the Gillespie algorithm. Furthermore, the validity of applying this algorithm was tested by comparing its results for large populations with those of the deterministic simulation.

### 3.1 Analysis of the influence of growth rates, catalysis rates and cost

According to Figure 1, it is observed that, for most parameter combinations, the probability of extinction of the population of cooperative replicators in the stochastic simulation is greater than in the deterministic simulation. This is because the total population is small and the initial population of cooperative replicators is even smaller, which increases the susceptibility to random effects [14].

Furthermore, in regions of the graphs where total extinction should occur according to the deterministic simulation, there is a probability, even if low, that the population of cooperative replicators will not go extinct in the stochastic simulation. Thus, the fact that the population is small can mean that the population of cooperative replicators does not go extinct for parameter values for which it would go extinct if the population were large [15].



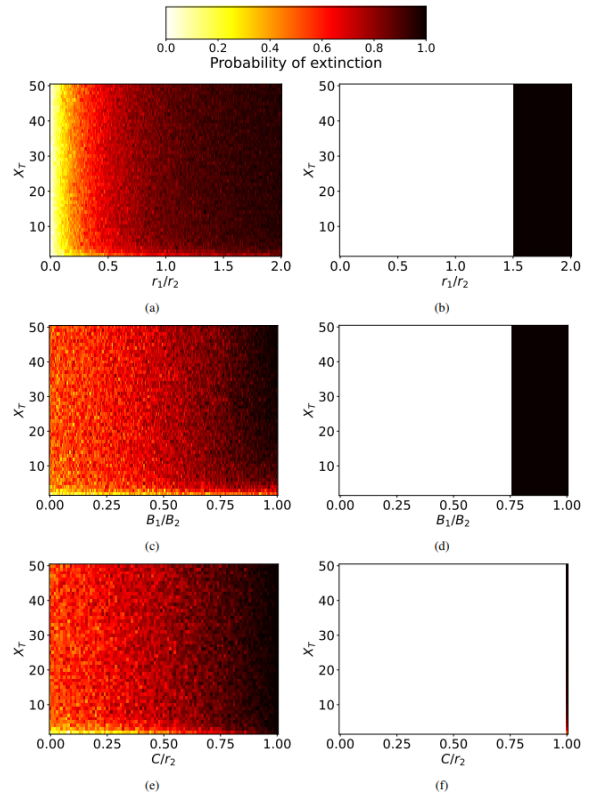
**Fig. 1** - Heat maps showing the relationship between the probability of extinction of the population of cooperative replicators and the growth rates  $r$ , the catalysis rates  $B$ , and the catalysis cost  $C$  suffered by cooperative replicators. Graphs (a), (c) and (e) were made from stochastic simulation, and (b), (d) and (f) from deterministic simulation. When not specified in the graphs, the values of the other parameters are:  $r_1 = r_2 = 1$ ,  $B_1 = 1$ ,  $B_2 = 2$ ,  $C = 0.5$ ,  $X_T = 10$ , and initial populations  $X_1 = 9$  and  $X_2 = 1$ .

### 3.2 Analysis of the influence of the total population

Based on Figure 2, something new can be observed in relation to the case of the previous section: for

small total populations, the probability of extinction of the population of cooperative replicators is lower, even for high values of  $\frac{r_1}{r_2}$ ,  $\frac{B_1}{B_2}$  and  $\frac{C}{r_2}$ , i.e., for unfavorable parameter values for cooperative replicators in large total populations.

For Figure 2 (a) and Figure 2 (e), this decrease in extinction probability occurs for very low values of  $X_T$ , which is because the initial population of cooperative replicators becomes a considerable fraction of the total population. However, in the case of Figure 2 (c), it is observed that this decrease in probability occurs for slightly higher values of  $X_T$ . This is because the effect of cooperation is highly sensitive to the size of the total population [16].

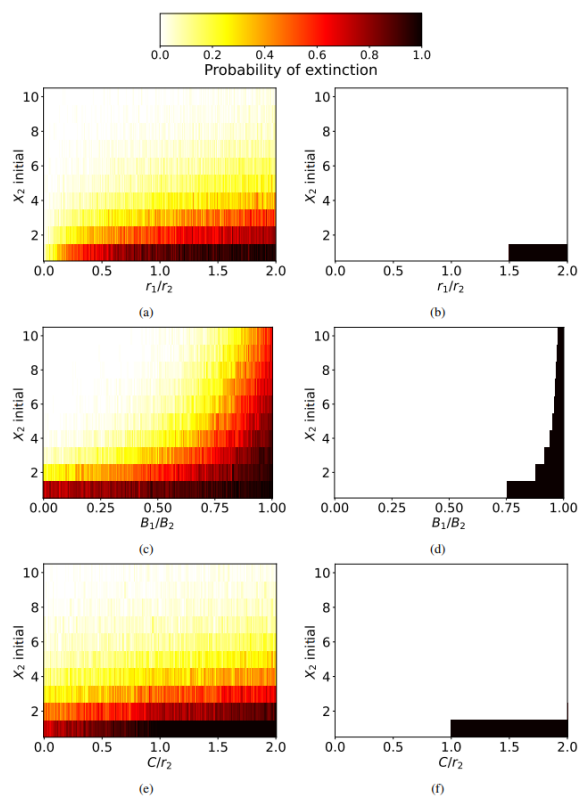


**Fig. 2** - Heat maps showing the relationship between the probability of extinction of the population of cooperative replicators and the total population  $X_T$ , the growth rates  $r$ , and the catalysis cost  $C$  suffered by cooperative replicators. Graphs (a), (c) and (e) were made from stochastic simulation, and (b), (d) and (f) from deterministic simulation. When not specified in the graphs, the values of the other parameters are:  $r_1 = r_2 = 1$ ,  $B_1 = 1$ ,  $B_2 = 2$ ,  $C = 0.5$ , and initial populations  $X_1 = 9$  and  $X_2 = 1$ .

### 3.3 Analysis of the influence of the initial population

When analyzing Figure 3 (c), it can be noted that, for values of  $\frac{B_1}{B_2}$  close to 1, the probability of extinction of the population of cooperative replicators decreases as the initial population increases, presenting a large difference in relation to the deterministic simulation, as shown in Figure 3 (d). This means that, for a small total population, the initial population of cooperative replicators has a large influence on the probability of

extinction of that population and the smaller the effect of cooperation, the larger the initial population will have to be to avoid extinction [17].



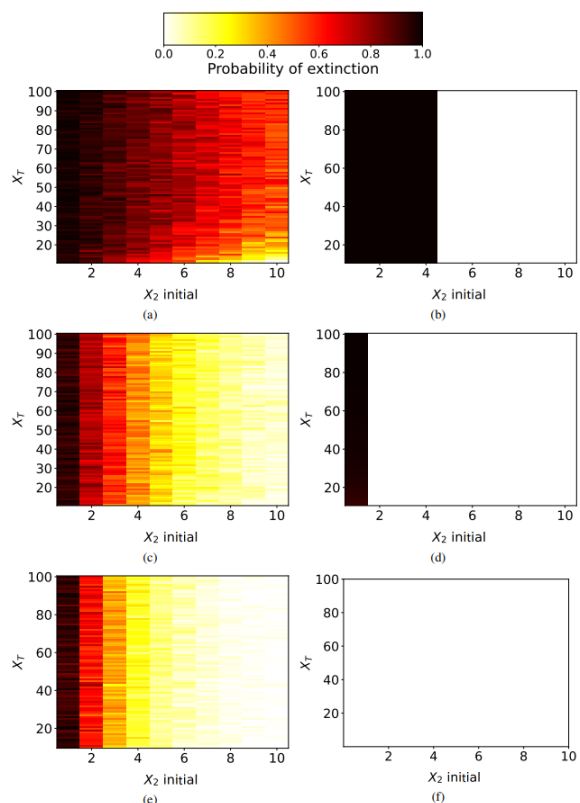
**Fig. 3** - Heat maps showing the relationship between the probability of extinction of the population of cooperative replicators and the initial population of cooperative replicators  $X_2$  initial, the growth rates  $r$ , the catalysis rate  $B$ , and the catalysis cost  $C$  suffered by cooperative replicators. Graphs (a), (c) and (e) were made from stochastic simulation, and (b), (d) and (f) from deterministic simulation. When not specified in the graphs, the values of the other parameters are:  $r_1 = r_2 = 1, B_1 = 1, B_2 = 2, C = 0.5$ , and  $X_T = 20$ .

### 3.4 Analysis of the relationship between total population and initial population

In Figure 4, it is observed that the greater the catalysis precision of cooperative replicators in relation to the catalysis of selfish replicators (the higher  $B_2$  is in relation to  $B_1$ ), the lower the value of the initial population of cooperative replicators for which the probability of extinction of this population becomes less than 50%, in addition to the faster decrease in the probability of extinction as the initial population of cooperative replicators  $X_2$  initial increases [18].

Furthermore, it can be noted that the greater the catalysis precision of cooperative replicators, the smaller the influence of the total population on the probability of extinction of the population of cooperative replicators for a given initial population. Thus, for a given initial population of cooperative replicators, the greater the catalysis precision, the lower the dependence of the probability of extinction of the population of cooperative replicators on the total population. This shows that the catalysis

precision of cooperative replicators is essential for small initial populations of these replicators to survive in large total populations [19].



**Fig. 4** - Heat maps showing the relationship between the probability of extinction of the population of cooperative replicators and the initial population of cooperative replicators  $X_2$  initial, and the total population  $X_T$ . Graphs (a), (c) and (e) were made from stochastic simulation, and (b), (d) and (f) from deterministic simulation. When not specified in the graphs, the values of the other parameters are:  $r_1 = r_2 = 1, B_1 = 1$ , and  $C = 0.5$ .

## 4. Conclusions

This study, focused on elucidating the introduction of cooperation in molecular evolution in a prebiotic context, revealed crucial aspects about the interaction between finite populations of selfish and cooperative replicators. Through a rigorous method that combined analytical and computational approaches (which involved both deterministic and stochastic simulations) to study the replicator equation model, significant results were obtained, emphasizing the sensitivity of cooperation to population size and precision of catalysis.

It was found that, in small total populations, susceptibility to random fluctuations is amplified, exerting a considerable influence on the survival of cooperative replicators. Notably, for small total populations, cooperative replicators may be more likely to avoid extinction, even under conditions that favor their elimination in larger populations. This phenomenon highlights the importance of population size in prebiotic evolution, an aspect often underestimated.

Another important conclusion is the decisive influence of the catalysis precision of cooperative replicators. The greater this precision, the smaller the initial population of cooperative replicators needed to reduce the risk of extinction below 50%. This result points to precise catalysis as a vital mechanism for the survival and prevalence of cooperative replicators, which emerged in small initial populations.

The original contribution of this research lies in the great diversity of analyzes obtained from the application of stochastic simulations, carried out using the Gillespie algorithm, in the replicator equation model. This approach allowed for a deeper understanding of stochastic dynamics in finite populations of replicators, complementing traditional analytical analyzes and deterministic simulations. Furthermore, this study resulted in conclusions similar to other studies with different approaches. Therefore, this research offers a comprehensive analysis of the interactions between prebiotic replicators that allowed cooperation to be introduced into molecular evolution.

## 5. Acknowledgement

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